

LAPTH-833/01
DTP-MSU/01-03

Bertotti-Robinson type solutions to Dilaton-Axion Gravity

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6 February 2001

Abstract

We present a new solution to dilaton-axion gravity which looks like a rotating Bertotti-Robinson (BR) Universe. It is supported by an homogeneous Maxwell field and a linear axion and can be obtained as a near-horizon limit of extremal rotating dilaton-axion black holes. It has the isometry $SL(2, R) \times U(1)$ where $U(1)$ is the remnant of the $SO(3)$ symmetry of BR broken by rotation, while $SL(2, R)$ corresponds to the AdS_2 sector which no longer factors out of the full spacetime. Alternatively our solution can be obtained from the $D = 5$ vacuum counterpart to the dyonic BR with equal electric and magnetic field strengths. The derivation amounts to smearing it in $D = 6$ and then reducing to $D = 4$ with dualization of one Kaluza-Klein two-form in $D = 5$ to produce an axion. Using a similar dualization procedure, the rotating BR solution is uplifted to $D = 11$ supergravity. We show that it breaks all supersymmetries of $N = 4$ supergravity in $D = 4$, and that its higher dimensional embeddings are not supersymmetric either. But, hopefully it may provide a new arena for conformal mechanics and holography. Applying a complex coordinate transformation we also derive a BR solution endowed with a NUT parameter.

PACS no: 04.20.Jb, 04.50.+h, 46.70.Hg

1 Introduction

The discovery of AdS/CFT dualities [1] (for a review see [2]) stimulated search for geometries containing AdS sectors. Recall that AdS geometry typically arises as the near horizon (throat) limit of static charged BPS black holes and/or p-branes in various dimensions. The known examples of AdS/CFT correspondence make use of geometries $AdS_n \times K$ with K being some compact manifold. For rotating black holes/branes the near horizon limit generically is different: one finds a non-trivial mixing of AdS_n and K . Nevertheless, the asymptotic geometry relevant

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for holography may remain unaffected by rotation for $n \geq 3$, so the ADS/CFT correspondence applies directly. This is not so in the case of AdS_2 [3, 4]. While for non-rotating extremal charged black holes the near-horizon geometry factorizes as $AdS_2 \times S^2$, in the rotating case the azimuthal coordinates mixes with the time coordinate in such a way that the $AdS_2 \times S^2$ geometry is not recovered asymptotically. Therefore, in higher dimensions the rotation parameter just adds a specific excitation mode in the dual theory [5, 6, 7, 8], but in four dimensions it apparently leads to more serious consequences, whose nature is not clear yet. Additional problems in the four-dimensional case are related to the fact that the AdS_2 holography is less well understood than the higher-dimensional examples [9, 10, 11, 12, 13] (for some recent progress in this direction see [14]).

Bardeen and Horowitz [3] observed that violation of the direct product structure $AdS_2 \times K$ due to rotation in $D = 4$ is manifest already for vacuum Kerr black holes. In the throat geometry of the extreme Kerr (and Kerr-Newman) four-dimensional black hole one finds the mixing of azimuthal and time coordinates which does not disappear in the asymptotic region, but grows infinitely instead leading to the singular nature of the conformal boundary. Nevertheless, the geometry still share some important features with $AdS_n \times K$ spacetimes such as (partial) confinement of timelike geodesics and discreteness of the Klein-Gordon particle spectrum on the geodesically complete AdS patch. But, apart from the fact that the near-horizon spacetime is no longer the direct product of AdS_2 with something, the geometry is also plagued by cumbersome θ -depending factors which modify the spectrum of the angular Laplacian. It turns out that the spectrum of the Klein-Gordon field contains a continuous sector which exhibits superradiance inherited from Kerr. All this substantially complicates the analysis and no definitive conclusion was gained in [3] about the possible relevance of such geometries in holography.

Here we present another geometry containing the AdS_2 sector mixed non-trivially with the rest of the spacetime which has the advantage of not being afflicted by θ -factors. It can be obtained as the near-horizon limit of extremal rotating dilaton-axion black holes (solutions to the Einstein-Maxwell-dilaton-axion (EMDA) theory). This is therefore a non-vacuum solution which is supported by a homogeneous Maxwell field (similarly to the Bertotti-Robinson (BR) spacetime) and a linear axion. The rotation breaks the $SO(3)$ symmetry of BR so that the solution is only axially symmetric. Meanwhile, as in the Bardeen-Horowitz case, the $SL(2, R)$ symmetry of the AdS component still holds, so the full isometry group is $SL(2, R) \times U(1)$.

We show that this geometry has a non-trivial connection with BR via a non-local dimensional reduction mechanism [15] involving dualization of Kaluza-Klein two-forms in order to generate higher-rank antisymmetric forms. Starting with the dyonic $D = 4$ BR with equal strengths of the electric and magnetic components one finds its purely vacuum $D = 5$ counterpart (the KK dilaton is not excited in the symmetric dyon case). This solution can be smeared into the sixth dimension providing the ‘BR6’ vacuum geometry. Then one performs dimensional reduction back to five dimensions dualizing the Kaluza-Klein two-form and reinterpreting the resulting three-form as a NS-NS field. Finally going to four dimensions via the usual KK reduction one recovers the EMDA theory counterpart of the BR6 which coincides with our near-horizon limit of rotating EMDA black holes.

Using the same mechanism of generation of antisymmetric forms, we uplift the new solution into eleven-dimensional supergravity where it is supported by a non-trivial four-form. This is based on the correspondence between eight-dimensional vacuum gravity with two commuting Killing vector fields and a consistent $2 + 3 + 6$ three-block truncation of $D = 11$ supergravity [16]. To apply this technique one has first to smear BR6 in two additional dimensions and then

use dualization of the KK two-form in six dimensions to get the four-form which is then used to reconstruct the four-form of $D = 11$ supergravity. In doing this there are two options in the choice of the Killing vectors which lead to two supergravity solutions with different four-forms but the same metric.

Since rotation breaks the BPS condition for extremal dilaton-axion black holes, it can be expected that our rotating BR solution is not supersymmetric in the sense of $D = 4$, $\mathcal{N} = 4$ supergravity. To check this, we consider the purely algebraic equation for variation of dilatino and show that this variation is non-zero. We also check by a direct computation that the $D = 11$ embedding of our solution is not supersymmetric in the sense of $D = 11$ supergravity. Nevertheless we argue that the rotating BR geometry provides an interesting new arena for conformal mechanics and holography.

2 Near-horizon limit of rotating EMDA black holes

Consider the Einstein-Maxwell-Dilaton-Axion (EMDA) theory which is a truncated version of the bosonic sector of $D = 4$, $\mathcal{N} = 4$ supergravity. The action describes the gravity-coupled system of two scalar fields: dilaton ϕ and (pseudoscalar) axion κ , and an Abelian vector field A_μ :

$$S = \frac{1}{16\pi} \int d^4x \sqrt{|g|} \left\{ -R + 2\partial_\mu \phi \partial^\mu \phi + \frac{1}{2} e^{4\phi} \partial_\mu \kappa \partial^\mu \kappa - e^{-2\phi} F_{\mu\nu} F^{\mu\nu} - \kappa F_{\mu\nu} \tilde{F}^{\mu\nu} \right\}, \quad (2.1)$$

where¹ $\tilde{F}^{\mu\nu} = \frac{1}{2} E^{\mu\nu\lambda\tau} F_{\lambda\tau}$, $F = dA$. The black hole solutions to this theory were extensively studied in the recent past [17, 18, 19, 20, 21, 22]. These depend on six real parameters, the complex mass $\mathcal{M} = M + iN$ (where N is the NUT parameter), electromagnetic charge $\mathcal{Q} = Q + iP$ and axion-dilaton charge $\mathcal{D} = D + iA$ constrained by $\mathcal{D} = -\mathcal{Q}^2/2\mathcal{M}$, and the rotation parameter a . The general black hole metric is of the form

$$ds^2 = \frac{\Delta - a^2 \sin^2 \theta}{\Sigma} (dt - \omega d\varphi)^2 - \Sigma \left(\frac{dr^2}{\Delta} + d\theta^2 + \frac{\Delta \sin^2 \theta}{\Delta - a^2 \sin^2 \theta} d\varphi^2 \right), \quad (2.2)$$

with

$$\begin{aligned} \Delta &= (r - r_-)(r - 2M) + a^2 - (N - N_-)^2, \\ \Sigma &= r(r - r_-) + (a \cos \theta + N)^2 - N_-^2, \\ \omega &= \frac{2}{a^2 \sin^2 \theta - \Delta} [N \Delta \cos \theta + a \sin^2 \theta (M(r - r_-) + N(N - N_-))], \end{aligned} \quad (2.3)$$

and

$$r_- = \frac{M|\mathcal{Q}|^2}{|\mathcal{M}|^2}, \quad N_- = \frac{N}{2M} r_-. \quad (2.4)$$

The vector field may be parametrized by two scalar electric (v) and magnetic (u) potentials defined by

$$\begin{aligned} F_{i0} &= \partial_i v / \sqrt{2}, \\ e^{-2\phi} F^{ij} + \kappa \tilde{F}^{ij} &= (\Sigma \sin \theta)^{-1} \epsilon^{ijk} \partial_k u / \sqrt{2}. \end{aligned} \quad (2.5)$$

¹Here $E^{\mu\nu\lambda\tau} \equiv |g|^{-1/2} \epsilon^{\mu\nu\lambda\tau}$, with $\epsilon^{1234} = +1$, where $x^4 = t$ is the time coordinate.

The potentials v and u and the axion-dilaton field are given (after adapting the formulas of [19] to the conventions of [20]) by

$$\begin{aligned} v &= \sqrt{2} \frac{e^{\phi_\infty}}{\Sigma} \operatorname{Re}[\mathcal{Q}(r - r_- - i\delta)], \\ u &= \sqrt{2} \frac{e^{\phi_\infty}}{\Sigma} \operatorname{Re}[\mathcal{Q}z_\infty(r - r_- - i\delta)], \\ z &\equiv \kappa + i e^{-2\phi} = \frac{z_\infty \rho + \mathcal{D}^* z_\infty^*}{\rho + \mathcal{D}^*}, \end{aligned} \quad (2.6)$$

where

$$\delta = a \cos \theta + N - N_-, \quad \rho = r - \frac{\mathcal{M}r_-}{2M} - i\delta, \quad (2.7)$$

and the (physically irrelevant) asymptotic value of the axion-dilaton field $z_\infty \equiv \kappa_\infty + i e^{-2\phi_\infty}$ will be chosen later for convenience.

The metric (2.2) has two horizons located at the zeroes $r = r_H^\pm$ of the function Δ :

$$r_H^\pm = M + r_-/2 \pm \sqrt{(|\mathcal{M}| - |\mathcal{D}|)^2 - a^2}. \quad (2.8)$$

The extremal solutions correspond to the case

$$|\mathcal{D}| = |\mathcal{M}| - a \quad (2.9)$$

where these two horizons coincide, with $\Sigma > 0$ (without loss of generality we assume $a > 0$). In this case, using

$$r - r_- = r - r_H + a \frac{M}{|\mathcal{M}|}, \quad (2.10)$$

we can rewrite the metric as

$$ds^2 = \frac{\Sigma \Delta \sin^2 \theta}{\Gamma} dt^2 - \Sigma \left(\frac{dr^2}{\Delta} + d\theta^2 \right) - \frac{\Gamma}{\Sigma} (d\varphi - \Omega dt)^2 \quad (2.11)$$

with

$$\begin{aligned} \Gamma &= \Delta [(\eta + 4\gamma) \sin^2 \theta - 4N^2 \cos^2 \theta] + 4(\gamma - aN \cos \theta)^2 \sin^2 \theta, \\ \Sigma &= \eta + 2\gamma, \quad \Omega = 2(N\eta \cos \theta + a\gamma \sin^2 \theta)/\Gamma, \\ \Delta &= (r - r_H)^2, \quad \eta = \Delta - a^2 \sin^2 \theta, \quad \gamma = M(r - r_H) + a(|\mathcal{M}| + N \cos \theta). \end{aligned} \quad (2.12)$$

On the horizon $r = r_H$, the metric functions occurring in (2.12) simplify to

$$\begin{aligned} \Gamma_H &= 4a^2 |\mathcal{M}|^2 \sin^2 \theta, \quad \Omega_H = 1/2 |\mathcal{M}|, \\ \Sigma_H &= 2a(|\mathcal{M}| + N \cos \theta) - a^2 \sin^2 \theta. \end{aligned} \quad (2.13)$$

It follows that in the static case $a = 0$, the horizon reduces to a point, so that the question of near-horizon limit becomes meaningless. But for rotating extremal black holes $a \neq 0$ one finds a non-trivial limiting solution. In this case, before taking the near-horizon limit, let us first transform to a frame co-rotating with the horizon:

$$\overline{\varphi} \equiv \varphi - \Omega_H t. \quad (2.14)$$

In this frame, the differential angular velocity is, near the horizon,

$$\overline{\Omega} = \Omega - \Omega_H = -\frac{2aM(r - r_H)\sin^2\theta}{\Gamma} + O(\Delta). \quad (2.15)$$

Let us now put

$$r - r_H \equiv \lambda x, \quad \cos\theta \equiv y, \quad t \equiv \frac{r_0^2}{\lambda} \bar{t} \quad (r_0^2 = 2a|\mathcal{M}|), \quad (2.16)$$

and take the limit $\lambda \rightarrow 0$. In this limit the extreme metric in the rotating frame reduces to

$$ds^2 = r_0^2 [(\alpha + \nu y + \beta y^2)(x^2 dt^2 - \frac{dx^2}{x^2}) - \frac{\alpha + \nu y + \beta y^2}{1 - y^2} dy^2 - \frac{1 - y^2}{\alpha + \nu y + \beta y^2} (d\varphi + \mu x dt)^2], \quad (2.17)$$

where $\beta = a/2|\mathcal{M}|$, $\alpha = 1 - \beta$, $\mu = M/|\mathcal{M}|$, $\nu = N/|\mathcal{M}|$ ($\mu^2 + \nu^2 = 1$), and we have relabelled the coordinates $(\bar{t}, \bar{\varphi}) \rightarrow (t, \varphi)$.

Just as the extreme Kerr or Kerr-Newman geometries in Einstein-Maxwell theory [3], the near-horizon geometry (2.21) admits four Killing vectors

$$\begin{aligned} L_1 &= x\partial_x - t\partial_t, \\ L_2 &= xt\partial_x - \frac{1}{2}(x^{-2} + t^2)\partial_t + \mu x^{-1}\partial_\varphi, \\ L_3 &= \partial_t, \\ L_4 &= \partial_\varphi, \end{aligned} \quad (2.18)$$

generating the group $SL(2, R) \times U(1)$. Indeed, the metric (2.17) becomes, in the NUT-less case $\nu = 0$,

$$ds^2 = r_0^2 [(\alpha + \beta y^2)(x^2 dt^2 - \frac{dx^2}{x^2}) - \frac{\alpha + \beta y^2}{1 - y^2} dy^2 - \frac{1 - y^2}{\alpha + \beta y^2} (d\varphi + x dt)^2]. \quad (2.19)$$

This is similar in form to the extreme Kerr-Newman near-horizon metric [3, 23]

$$ds_{EM}^2 = r_{0EM}^2 [(\alpha_{EM} + \beta_{EM} y^2)(x^2 dt^2 - \frac{dx^2}{x^2}) - \frac{\alpha_{EM} + \beta_{EM} y^2}{1 - y^2} dy^2 - \frac{1 - y^2}{\alpha_{EM} + \beta_{EM} y^2} (d\varphi + \mu_{EM} x dt)^2] \quad (2.20)$$

with $r_{0EM}^2 = M^2 + a^2$, $\beta_{EM} = a^2/(M^2 + a^2)$, $\alpha_{EM} = 1 - \beta_{EM}$, $\mu_{EM} = 2aM/(M^2 + a^2)$, where $M^2 = a^2 + Q^2$. The two extreme geometries (2.19) and (2.20) become identical in the neutral case $Q = 0$ ($M = a$) and reduce to the extreme Kerr geometry with $\alpha = \beta = 1/2$.

Now let us take the limit $a \rightarrow 0$ in the NUT-less near-horizon geometry (2.19), while keeping $r_0^2 = 2aM$ fixed. In this manner we arrive at the metric

$$ds^2 = x^2 dt^2 - \frac{dx^2}{x^2} - \frac{dy^2}{1 - y^2} - (1 - y^2)(d\varphi + x dt)^2 \quad (2.21)$$

(we have scaled r_0^2 to unity). This metric is remarkably similar in form to the Bertotti-Robinson metric (the near-horizon geometry of the extreme Reissner-Nordström black hole, i.e. the static limit $a \rightarrow 0$ of (2.20)),

$$ds^2 = x^2 dt^2 - \frac{dx^2}{x^2} - \frac{dy^2}{1-y^2} - (1-y^2) d\varphi^2. \quad (2.22)$$

However, unlike the BR metric, the BREMDA metric (2.21) is non-static, and invariant only under the group $SL(2, R) \times U(1)$. The t, x coordinates do not cover the full AdS hyperboloid. The geodesically complete manifold is covered by another coordinate patch in which case the metric reads (we preserve the same symbols for radial and azimuthal coordinates)

$$ds^2 = (1+x^2) d\tau^2 - \frac{dx^2}{1+x^2} - \frac{dy^2}{1-y^2} - (1-y^2) (d\varphi + x d\tau)^2. \quad (2.23)$$

Another useful coordinate system (also incomplete) is given by

$$ds^2 = (x^2 - 1) d\tau^2 - \frac{dx^2}{x^2 - 1} - d\theta^2 - \frac{dy^2}{1-y^2} - (1-y^2) (d\varphi + x d\tau)^2. \quad (2.24)$$

Now proceed along the same lines with the matter fields. From the last Eq. (2.6), the dilaton and axion fields reduce on the horizon to

$$\begin{aligned} e^{-2\phi_H} &= e^{-2\phi_\infty} \frac{2a(|\mathcal{M}| + Ny) - a^2(1-y^2)}{(M+D)^2 + (N+A+ay)^2}, \\ \kappa_H &= \kappa_\infty + 2e^{-2\phi_\infty} \frac{D(N+ay) - AM}{(M+D)^2 + (N+A+ay)^2}. \end{aligned} \quad (2.25)$$

For $N = 0$ and in the limit $a \rightarrow 0$, these become (after neglecting terms of order a^2)

$$\begin{aligned} e^{-2\phi_H} &= e^{-2\phi_\infty} \frac{aM}{P^2}, \\ \kappa_H &= \kappa_\infty - e^{-2\phi_\infty} \frac{M}{P^2} (A - ay). \end{aligned} \quad (2.26)$$

Now we choose for convenience

$$z_\infty = -2PQ^*/r_0^2 \quad (2.27)$$

($r_0^2 = 2aM$), leading to the BREMDA dilaton and axion fields

$$\phi_H = 0, \quad \kappa_H = \cos \theta, \quad (2.28)$$

irrespective of the original values of Q and P (provided $P \neq 0$).

The determination of the near-horizon behavior of the gauge field is more involved. With the choice (2.27), the scalar potentials for the NUT-less extreme black hole are

$$\begin{aligned} v &= \sqrt{2} \frac{e^{\phi_\infty}}{\Sigma} (Q(r - r_H + a) + Pa \cos \theta), \\ u &= -\sqrt{2} \frac{e^{-\phi_\infty}}{\Sigma} \frac{2M(M-a)}{P} (r - r_H + a). \end{aligned} \quad (2.29)$$

First, we must transform these to the rotating and rescaled coordinate frame $(\bar{\varphi}, \bar{t})$

$$\begin{pmatrix} d\bar{\varphi} \\ d\bar{t} \end{pmatrix} = \begin{pmatrix} 1 & -\Omega_H \\ 0 & \lambda/r_0^2 \end{pmatrix} \begin{pmatrix} d\varphi \\ dt \end{pmatrix}. \quad (2.30)$$

From (2.5) we obtain the transformation laws

$$\begin{aligned} \partial_i \bar{v} &= \frac{r_0^2/\lambda}{\Sigma^2 \Delta \sin^2 \theta / \Gamma^2 - \Omega^2} \left\{ \left(\frac{(\Sigma^2 \Delta \sin^2 \theta)}{\Gamma^2} - \Omega \bar{\Omega} \right) \partial_i v \right. \\ &\quad \left. + \frac{\Omega_H e^{2\phi} \Delta \sin \theta}{\Gamma} g_{ij} \epsilon^{jk} (\partial_k u - \kappa \partial_k v) \right\}, \\ \partial_i \bar{u} &= \frac{r_0^2/\lambda}{\Sigma^2 \Delta \sin^2 \theta / \Gamma^2 - \Omega^2} \left\{ \left(\frac{(\Sigma^2 \Delta \sin^2 \theta)}{\Gamma^2} - \Omega \bar{\Omega} \right) \partial_i u \right. \\ &\quad \left. - \frac{\Omega_H e^{2\phi} \Delta \sin \theta}{\Gamma} g_{ij} \epsilon^{jk} ((e^{-4\phi} + \kappa^2) \partial_k v - \kappa \partial_k u) \right\} \end{aligned} \quad (2.31)$$

$(i, j = r, \theta)$. Then, using

$$\partial_r \Sigma \simeq 2M, \quad \partial_\theta \Sigma = -2a^2 \sin \theta \cos \theta, \quad (2.32)$$

we evaluate the derivatives in (2.31) near the horizon, keeping only the leading terms in a (only the partial derivatives relative to θ contribute in this order):

$$\begin{aligned} \partial_r \bar{v}_H &\simeq -\frac{r_0}{\lambda} \cos \theta, & \partial_\theta \bar{v}_H &\simeq \frac{r_0}{\lambda} (r - r_H) \sin \theta, \\ \partial_r \bar{u}_H &\simeq \frac{r_0}{\lambda} \sin^2 \theta, & \partial_\theta \bar{u}_H &\simeq \frac{r_0}{\lambda} (r - r_H) 2 \sin \theta \cos \theta. \end{aligned} \quad (2.33)$$

From these we obtain the near-horizon potentials

$$\begin{aligned} \bar{v}_H &= -r_0 x \cos \theta, \\ \bar{u}_H &= r_0 x \sin^2 \theta, \end{aligned} \quad (2.34)$$

again irrespective of the original values of the electric and magnetic charges. We have checked that these, together with the near-horizon metric (2.21) and axion-dilaton fields (2.28) solve the field equations as given in [20]. From the potentials (2.34), after setting r_0 to unity, we recover the near-horizon gauge fields

$$\begin{aligned} F_{14} &= -\frac{y}{\sqrt{2}}, & F_{23} &= -\frac{1}{\sqrt{2}}, & F_{24} &= -\frac{x}{\sqrt{2}}, \\ F^{13} &= -\frac{xy}{\sqrt{2}}, & F^{14} &= \frac{y}{\sqrt{2}}, & F^{23} &= -\frac{1}{\sqrt{2}} \end{aligned} \quad (2.35)$$

(with $x^1 = x$ and $x^2 = y = \cos \theta$), deriving from the gauge potentials $A_3 = -y/\sqrt{2}$, $A_4 = -xy/\sqrt{2}$. Finally, passing to more general coordinates containing a free parameter b , we can write the BREMDA solution as follows²

$$\begin{aligned} ds^2 &= (x^2 + b) d\tau^2 - \frac{dx^2}{x^2 + b} - d\theta^2 - \sin^2 \theta (d\varphi + x d\tau)^2, \\ A &= A_\mu dx^\mu = -\frac{\cos \theta}{\sqrt{2}} (d\varphi + x d\tau), \quad \kappa = \cos \theta. \end{aligned} \quad (2.36)$$

²Let us here mention that in [24] the low-energy limit of a certain conformal field theory was shown to correspond to a formal near-horizon limit of Kerr-NUT solutions of EMDA, with metric and matter fields different from (2.36). However one can show that these fields (given in Eq. (3.4) of [24]) do not solve the field equations of dilaton-axion gravity.

For $b = 0$ this coincides with the solution derived above by the limiting procedure (Poincaré coordinates on AdS sector), for positive non-zero b (usually set to $b = 1$) one has a coordinate patch covering the full AdS hyperboloid. For comparison consider the near-horizon limit of the Kerr solution [3]. Setting in (2.19) $r_0 = 1, \alpha = \beta = 1/2$ and passing to similar coordinates, we obtain

$$ds^2 = \frac{1}{2}(1 + \cos^2 \theta) \left[(x^2 + b) d\tau^2 - \frac{dx^2}{(x^2 + b)} - d\theta^2 \right] - \frac{2 \sin^2 \theta}{1 + \cos^2 \theta} (d\phi + x d\tau)^2. \quad (2.37)$$

In both cases the mixing of the azimuthal and time coordinates does not vanish as $x \rightarrow \infty$. Both metrics coincide in the equatorial plane, but differ for $\theta \neq \pi/2$. The BREMDA geometry is simpler due to the absence of cumbersome angular factors, and apparently is more suitable for the search of a holographic dual. We return to this question in a separate publication [25], while here we discuss other geometrical aspects of the new solution related to its embeddings in higher dimensions.

3 $D = 4$ EMDA from $D = 6$ Einstein gravity

Let us now show how $D = 4$ EMDA theory can be derived from the purely vacuum Einstein theory in six dimensions. This may be hinted from the following considerations. Dimensional reduction of stationary $D = 4$ EMDA to three dimensions leads to a gravity coupled sigma-model with the target space isometry group $Sp(4, R)$ [26, 27], while dimensional reduction of the 6D vacuum gravity to three dimensions gives a sigma model with the $SL(4, R)$ target space symmetry [28]. It was shown in [15] that a consistent truncation of the $SL(4, R)$ sigma model to the $Sp(4, R)$ one exists, i.e. any stationary solution of $D = 4$ EMDA gravity has a $D = 6$ vacuum gravity counterpart and vice versa. Here we show that this holds not only for stationary, but for any solutions of two theories. This duality is essentially non-local: it involves dualization of the Kaluza-Klein two-form in the intermediate five dimensions.

Let us start with the action

$$S = - \int d^6 x \sqrt{|g_6|} R_6, \quad (3.1)$$

denoting the 6-dimensional coordinates as x^μ, η, χ , and make the assumption of two commuting spacelike Killing vectors $\partial_\eta, \partial_\chi$. In any number of dimensions, the Kaluza-Klein dimensional reduction

$$ds_{n+1}^2 = e^{-2c\hat{\phi}} ds_n^2 - e^{2(n-2)c\hat{\phi}} (d\eta + \hat{C}_\mu dx^\mu)^2 \quad (3.2)$$

gives

$$\sqrt{|g_{n+1}|} R_{n+1} = \sqrt{|g_n|} [R_n - (n-1)(n-2)c^2 (\partial\hat{\phi})^2 + \frac{1}{4} e^{2(n-1)c\hat{\phi}} F^2(\hat{C}) + 2c \nabla^2 \hat{\phi}]. \quad (3.3)$$

For $n = 5, c = 1/\sqrt{6}$, this leads, after dualizing the 2-form $F(\hat{C})$ to a 3-form $\hat{H} = d\hat{K}$,

$$F^{\mu\nu}(\hat{C}) = - \frac{1}{6\sqrt{|g_5|}} e^{-\alpha\hat{\phi}} \epsilon^{\mu\nu\lambda\rho\sigma} \hat{H}_{\lambda\rho\sigma}, \quad (3.4)$$

to the reduced action for 6-dimensional sourceless gravity

$$S_5 = \int d^5 x \sqrt{|g_5|} [-R_5 + 2(\partial\hat{\phi})^2 + \frac{1}{12} e^{-\alpha\hat{\phi}} \hat{H}^2], \quad (3.5)$$

with $\alpha = 4\sqrt{2/3}$.

In (3.5) we recognize the action of 5-dimensional gravity coupled to a dilaton and a 3-form, as written down in [15]. In this paper it was observed that, under the assumption of two commuting Killing vectors ∂_4 and ∂_5 , this theory reduces to a 3-dimensional σ model with the $SL(4, R)$ symmetry group. Here we see that this symmetry follows directly from the $SL(4, R)$ symmetry of sourceless $D = 6$ gravity with 3 Killing vectors, which is a special case of sourceless n -dimensional gravity with $(n - 3)$ Killing vectors, as discussed by Maison [28].

In a second step, the 5-dimensional theory (3.5) with a spacelike Killing vector ∂_χ is further reduced by the Kaluza-Klein ansatz

$$\begin{aligned} ds_5^2 &= e^{-2\sigma/\sqrt{3}} ds_4^2 - e^{4\sigma/\sqrt{3}} (d\chi + D_\mu dx^\mu)^2, \\ \hat{K} &= K_{\mu\nu} dx^\mu \wedge dx^\nu + E_\mu d\chi \wedge dx^\mu, \end{aligned} \quad (3.6)$$

to the action

$$\begin{aligned} S_4 &= \int d^4x \sqrt{|g_4|} [-R_4 + 2(\partial\phi)^2 + (\partial\psi)^2 + \frac{1}{2}e^{4\phi}(\partial\kappa)^2 - \frac{1}{4}e^{2(\psi-\phi)}F^2(D) \\ &\quad - \frac{1}{4}e^{-2(\psi+\phi)}F^2(E) - \frac{\kappa}{4}(F(D)\tilde{F}(E) + F(E)\tilde{F}(D))], \end{aligned} \quad (3.7)$$

where

$$\phi = \sqrt{\frac{2}{3}}\hat{\phi} - \sqrt{\frac{1}{3}}\sigma, \quad \psi = \sqrt{2}\left(\sqrt{\frac{1}{3}}\hat{\phi} + \sqrt{\frac{2}{3}}\sigma\right), \quad (3.8)$$

and κ is the dual of the 3-form

$$H \equiv dK - D \wedge F(E) = -e^{4\phi} * d\kappa. \quad (3.9)$$

Remarkably, the equations of motion for the fields D , E and ψ deriving from the action (3.7)

$$\begin{aligned} \nabla_\mu(e^{2(\psi-\phi)}F^{\mu\nu}(D) + \kappa\tilde{F}^{\mu\nu}(E)) &= 0, \\ \nabla_\mu(e^{-2(\psi+\phi)}F^{\mu\nu}(E) + \kappa\tilde{F}^{\mu\nu}(D)) &= 0, \\ \nabla^2\psi + \frac{1}{4}e^{-2\phi}(e^{2\psi}F^2(D) - e^{-2\psi}F^2(E)) &= 0 \end{aligned} \quad (3.10)$$

are consistent with the ansatz

$$\psi = 0, \quad D_\mu = E_\mu \equiv \sqrt{2}A_\mu, \quad (3.11)$$

which reduces the action (3.7) to the action (2.1) of EMDA (this is similar to the reduction of five-dimensional Kaluza-Klein theory to self-dual Einstein-Maxwell theory).

This two-step reduction of 6-dimensional vacuum gravity can be summarized in a direct reduction from 6 to 4 dimensions. From (3.4),

$$F_{\mu 5}(\hat{C}) = \partial_\mu \hat{C}_5 = -e^{-4\phi}\tilde{H}_\mu = \partial_\mu \kappa, \quad (3.12)$$

where $\mu = 1, \dots, 4$, so that the 5-dimensional 1-form \hat{C} reduces according to

$$\hat{C} = C_\mu dx^\mu + \kappa d\chi. \quad (3.13)$$

The two successive Kaluza-Klein ansätze (3.2) (for $n = 5$, $c = 1/\sqrt{6}$) and (3.6) can be combined into

$$ds_6^2 = e^{-\psi} ds_4^2 - e^{\psi-2\phi} (d\chi + D_\mu dx^\mu)^2 - e^{\psi+2\phi} (d\eta + C_\mu dx^\mu + \kappa d\chi)^2. \quad (3.14)$$

Finally, we compute

$$\begin{aligned} F_{\mu\nu}(C) &= -\frac{1}{2}\sqrt{|g_5|} e^{-\alpha\hat{\phi}} \epsilon_{\mu\nu\lambda\rho 5} \hat{H}^{\lambda\rho 5} \\ &= \frac{1}{2}\sqrt{|g_4|} \epsilon_{\mu\nu\lambda\rho} (e^{-2(\psi+\phi)} F^{\lambda\rho}(E) - e^{-4\phi} D_\tau H^{\tau\lambda\rho}) \\ &= F_{\mu\nu}(B) + F_{\mu\nu}(\kappa D), \end{aligned} \quad (3.15)$$

with the definition (solving the second equation (3.10))

$$F_{\mu\nu}(B) \equiv e^{-2(\psi+\phi)} \tilde{F}_{\mu\nu}(E) - \kappa F_{\mu\nu}(D). \quad (3.16)$$

Accordingly we can rewrite the double Kaluza-Klein ansatz (3.14) as

$$ds_6^2 = e^{-\psi} ds_4^2 - e^{\psi-2\phi} \theta^2 - e^{\psi+2\phi} (\zeta + \kappa\theta)^2, \quad (3.17)$$

with

$$\theta \equiv d\chi + D_\mu dx^\mu, \quad \zeta \equiv d\eta + B_\mu dx^\mu. \quad (3.18)$$

Taking into account (3.11), it follows that the ansatz for reducing 6-dimensional vacuum gravity to EMDA may be written

$$\begin{aligned} ds_6^2 &= ds_4^2 - e^{-2\phi} \theta^2 - e^{2\phi} (\zeta + \kappa\theta)^2, \\ \theta &\equiv d\chi + \sqrt{2}A_\mu dx^\mu, \quad \zeta \equiv d\eta + B_\mu dx^\mu, \\ F_{\mu\nu}(B) &\equiv \sqrt{2}(e^{-2\phi} \tilde{F}_{\mu\nu}(A) - \kappa F_{\mu\nu}(A)). \end{aligned} \quad (3.19)$$

4 $D = 6$ vacuum counterpart of BREMDA

Using the machinery of the preceding section, we can show that BREMDA is dual to the six-dimensional vacuum solution whose standard KK reduction gives the usual $D = 4$ dyonic BR solution with equal electric and magnetic strengths. From (2.28) and (2.35) we obtain (note that the coordinate transformation $t, x, y, \varphi \rightarrow t, x, \theta, \varphi$ reverses orientation, so that accordingly we must change the sign of the axion)

$$F_{14}(B) = 1 - y^2, \quad F_{23}(B) = -2y, \quad F_{24}(B) = -2xy, \quad (4.1)$$

leading (in a suitable gauge) to the 1-form

$$B = -y^2 d\varphi + x(1 - y^2) dt. \quad (4.2)$$

It follows that the 6-dimensional line element corresponding to (2.36) is (with $b = -c^2$)

$$\begin{aligned} ds_6^2 &= (x^2 - c^2) dt^2 - \frac{dx^2}{x^2 - c^2} - \frac{dy^2}{1 - y^2} - (1 - y^2)(d\varphi + x dt)^2 \\ &\quad - (d\chi - xy dt - y d\varphi)^2 - (d\eta + x dt - y d\chi)^2. \end{aligned} \quad (4.3)$$

This may be rearranged to the more compact form

$$ds_6^2 = (x^2 - c^2) dt^2 - \frac{dx^2}{x^2 - c^2} - \frac{dy^2}{1 - y^2} - (1 - y^2) d\chi^2 - (d\varphi + x dt - y d\chi)^2 - (d\eta + x dt - y d\chi)^2, \quad (4.4)$$

which is explicitly symmetric between the two Killing vectors ∂_φ and ∂_η , and enjoys the higher symmetry group $SL(2, R) \times SO(3) \times U(1) \times U(1)$.

Eq. (4.4) represents the Bertotti-Robinson solution of 6-dimensional vacuum gravity. A simpler form is achieved by making a $\pi/4$ rotation in the plane of the two Killing vectors $(\partial_\varphi, \partial_\eta)$ and relabelling the third spacelike Killing direction according to

$$d\varphi = -\frac{1}{\sqrt{2}}(d\bar{\eta} + d\bar{\chi}), \quad d\eta = \frac{1}{\sqrt{2}}(d\bar{\eta} - d\bar{\chi}), \quad d\chi = d\bar{\varphi}. \quad (4.5)$$

This leads to

$$ds_6^2 = (x^2 - c^2) dt^2 - \frac{dx^2}{x^2 - c^2} - \frac{dy^2}{1 - y^2} - (1 - y^2) d\bar{\varphi}^2 - (d\bar{\chi} - \sqrt{2}x dt + \sqrt{2}y d\bar{\varphi})^2 - d\bar{\eta}^2, \quad (4.6)$$

which is the trivial 6-dimensional embedding of the 5-dimensional Bertotti-Robinson metric

$$ds_5^2 = (x^2 - c^2) dt^2 - \frac{dx^2}{x^2 - c^2} - \frac{dy^2}{1 - y^2} - (1 - y^2) d\varphi^2 - (d\chi - \sqrt{2}x dt + \sqrt{2}y d\varphi)^2. \quad (4.7)$$

Remarkably, this is exactly the solution whose four-dimensional Kaluza-Klein reduction is the Einstein-Maxwell Bertotti-Robinson solution (2.22), for more details see Appendix A.

The metric (4.3) may also be dimensionally reduced relatively to the Killing vectors ∂_η and ∂_t (instead of ∂_χ). Choosing $c^2 = 1$, rearranging (4.3) as

$$ds_6^2 = (x^2 - 1) (d\varphi - y d\chi)^2 - \frac{dx^2}{x^2 - 1} - \frac{dy^2}{1 - y^2} - (1 - y^2) d\chi^2 - (dt + x d\varphi - xy d\chi)^2 - (d\eta + x dt - y d\chi)^2, \quad (4.8)$$

and relabelling the Killing directions according to $\varphi \rightarrow t \rightarrow \chi \rightarrow -\varphi$, one obtains the equivalent 6-dimensional metric

$$ds_6^2 = (x^2 - 1) (dt + y d\varphi)^2 - \frac{dx^2}{x^2 - 1} - \frac{dy^2}{1 - y^2} - (1 - y^2) d\varphi^2 - (d\chi + x dt + xy d\varphi)^2 - (d\eta + x d\chi + y d\varphi)^2. \quad (4.9)$$

Following (3.19), this may be reduced to the solution of EMDA:

$$ds_4^2 = (x^2 - 1) (dt + y d\varphi)^2 - \frac{dx^2}{x^2 - 1} - \frac{dy^2}{1 - y^2} - (1 - y^2) d\varphi^2, \\ A = \frac{x}{\sqrt{2}} (dt + y d\varphi), \quad \phi = 0, \quad \kappa = x, \quad (4.10)$$

with $B = -x^2 dt - (x^2 - 1) y d\varphi$ (κ may for instance be obtained by solving the Maxwell equations $dF(B) = 0$). This Bertotti-Robinson-NUT solution may be obtained from the BREMDA solution (2.36) by the correspondence (which is an isometry of the 5-dimensional Bertotti-Robinson metric (A.13))

$$t \leftrightarrow i\varphi, \quad x \leftrightarrow -y, \quad ds^2 \rightarrow -ds^2, \quad A \rightarrow -iA. \quad (4.11)$$

The dimensional reduction of (4.4) to (4.10) breaks the full symmetry group of the 6-dimensional Bertotti-Robinson solution to $SO(3) \times U(1)$.

Remarkably, this BR-NUT solution may also be obtained as a near-horizon limit of near-extremal static black hole solutions of EMDA with NUT charge. Such near-extremal black holes are defined by the condition that

$$(|\mathcal{M}| - |\mathcal{D}|)^2 \equiv |\mathcal{M}|^2 + |\mathcal{D}|^2 - |\mathcal{Q}|^2 \equiv \lambda^2 c^2 \quad (4.12)$$

is small. Putting $r - M - r_-/2 \equiv \lambda x$, we obtain for the metric functions in (2.2)

$$\Delta = \lambda^2(x^2 - c^2), \quad \Sigma = 2\lambda|\mathcal{M}| \left(\frac{M}{|\mathcal{M}|} x + c \right) + \Delta. \quad (4.13)$$

So the limit $\lambda \rightarrow 0$ will yield a Bertotti-Robinson-like metric only for $M = 0$. In this case, rescaling times by $t \rightarrow (r_0^2/\lambda)t$ as in (2.16), with now

$$r_0^2 = 2\lambda N, \quad (4.14)$$

we obtain the limiting 4-dimensional metric

$$ds^2 = r_0^2 \left[\frac{x^2 - c^2}{c} (dt + y d\varphi)^2 - \frac{c}{x^2 - c^2} (dx^2 + (x^2 - c^2) d\Omega^2) \right], \quad (4.15)$$

which is identical with (4.10) after scaling r_0^2 to unity and choosing without loss of generality $c = 1$ (one can always rescale $x \rightarrow cx$ and $ds^2 \rightarrow c ds^2$), as this construction goes along only for $c \neq 0$. Likewise, choosing

$$z_\infty = 2iQ\mathcal{Q}^*/cr_0^2, \quad (4.16)$$

we obtain from (2.6) (after reversing the signs of the pseudoscalars κ and u as explained above) the limiting (gauge transformed and rescaled) scalar potentials

$$\phi = 0, \quad \kappa = \frac{x}{c}, \quad v = \frac{r_0}{\sqrt{c}}x, \quad u = \frac{r_0}{\sqrt{c}} \frac{x^2 - c^2}{c}, \quad (4.17)$$

in agreement with (4.10). Again, we note that this limit is independent of the original values of Q and P , provided $Q \neq 0$.

5 Eleven-dimensional supergravity

The idea to generate higher rank antisymmetric forms by dualizing the KK two-forms was generalized to $D = 11$ supergravity as follows [16]. Starting with the lagrangian

$$S_{(11)} = \int d^{11}x \sqrt{-\hat{g}} \left\{ \hat{R}_{(11)} - \frac{1}{2 \times 4!} \hat{F}_{[4]}^2 \right\} - \frac{1}{6} \int \hat{F}_{[4]} \wedge \hat{F}_{[4]} \wedge \hat{A}_{[3]}, \quad (5.1)$$

we use the following three-block ansatz for the $D = 11$ metric

$$d\hat{s}_{(11)}^2 = g_2^{\frac{1}{2}} \delta_{ab} dz^a dz^b + g_3^{\frac{1}{3}} \delta_{ij} dy^i dy^j + (g_2 g_3)^{-\frac{1}{4}} g_{(6)\mu\nu} dx^\mu dx^\nu, \quad (5.2)$$

where all variables depend only on the six coordinates x^μ and $a, b = 1, 2$; $i, j = 1, 2, 3$; $\mu, \nu = 0, \dots, 5$. The three-form potential $A_{[3]}$ is reduced to its six-dimensional pull-back $B_{[3]}(x)$, the one-form $A_{[1]} = A_\mu dx^\mu$ and the scalar $\kappa(x)$, with

$$\hat{A}_{\mu z^1 z^2} = A_\mu(x), \quad \hat{A}_{y^1 y^2 y^3} = \kappa(x). \quad (5.3)$$

For the four-form one has

$$\hat{F}_{[4]} = G_{[4]} + F_{[2]}^2 \wedge Vol(2) + d\kappa \wedge Vol(3), \quad (5.4)$$

where $G_{[4]} = dB_{[3]}$, $F_{[2]} = dA_{[1]}$. After reduction to six dimensions we obtain the theory governed by the action

$$\begin{aligned} S_{(6)} = & \int d^6 x \sqrt{|g_{(6)}|} \left\{ R_{(6)} - \frac{e^{2\phi}}{2} (\nabla \kappa)^2 - \frac{1}{2} (\nabla \phi)^2 - \frac{3}{16} (\nabla \psi)^2 \right. \\ & \left. - \frac{1}{2} e^{-\phi} \left[\frac{1}{2!} e^{\frac{3}{4}\psi} F_{[2]}^2 + \frac{1}{4!} e^{-\frac{3}{4}\psi} G_{[4]}^2 \right] \right\} + \text{Chern-Simons terms}, \end{aligned} \quad (5.5)$$

where two new scalar fields are introduced via

$$\ln g_2 = \frac{2}{3} \phi - \psi, \quad \ln g_3 = -2\phi. \quad (5.6)$$

It is easy to see that κ, ϕ form a coset $SL(2, R)/SO(1, 1)$, while $F_{[2]}$ and the six-dimensional dual $\tilde{G}_{[2]} = *G_{[4]}$ (which is also a two-form) can be combined into the $SL(2, R)$ doublet

$$\mathcal{F}_{[2]} = e^{\psi/2} F_{[2]} + i e^{-\psi/2} \tilde{G}_{[2]}, \quad (5.7)$$

transforming under $SL(2, R)$ as follows

$$\begin{aligned} z & \rightarrow \frac{az + b}{cz + d}, \quad z = \kappa + i e^{-\phi}, \quad ad - bc = 1, \\ \mathcal{F}_{[2]} & \rightarrow (cz + d) \mathcal{F}_{[2]}, \quad \psi \rightarrow \psi + \text{const}. \end{aligned} \quad (5.8)$$

The multiplet of matter fields in the $D = 6$ action is the same as that which may be obtained from compactification of $D = 8$ vacuum gravity. Moreover, the action which follows from the $D = 8$ Einstein action with the metric ansatz

$$\begin{aligned} ds_{(8)}^2 & = g_{mn} \left(d\zeta^m + A_\mu^m dx^\mu \right) \left(d\zeta^n + A_\nu^n dx^\nu \right) + e^{-\frac{1}{4}\psi} g_{\mu\nu} dx^\mu dx^\nu, \\ e^\psi & = \det ||g_{mn}||, \end{aligned} \quad (5.9)$$

$(m, n = 1, 2)$ leads exactly to the theory (5.5) after the identification of variables

$$\begin{aligned} g_{mn} & = e^{\psi/2} \begin{pmatrix} e^\phi & \kappa e^\phi \\ \kappa e^\phi & e^{-\phi} + \kappa^2 e^\phi \end{pmatrix}, \\ dA_{[1]}^m & = F_{[2]}^m, \\ F_{[2]}^1 + \kappa F_{[2]}^2 & = e^{-\phi - \frac{3\psi}{4}} \tilde{G}_{[2]}. \end{aligned} \quad (5.10)$$

More precisely, the field equation for $B_{[3]}$ becomes a Bianchi identity for $A_{[1]}^1$ and vice versa. Note, that interchanging the $D = 8$ Killing vectors, i.e. relabelling $A_\mu^1 \leftrightarrow A_\mu^2$ we will get different $D = 11$ field configurations.

Now we can construct a solution to $D = 11$ supergravity which is dual to the eight-dimensional vacuum metric obtained from BR6(4.6) smeared in two flat extra dimensions

$$ds_8^2 = (x^2 - c^2) dt^2 - \frac{dx^2}{x^2 - c^2} - \frac{dy^2}{1 - y^2} - (1 - y^2) d\varphi^2 - (d\chi - \sqrt{2}x dt + \sqrt{2}y d\varphi)^2 - d\bar{\eta}^2 - dz_1^2 - dz_2^2. \quad (5.11)$$

This solution possesses several commuting Killing vectors, from which one can choose any pair to be used in KK reduction back to six dimensions.

Choosing $\zeta_1 = \chi, \zeta_2 = \eta$, we have

$$A^1 = \sqrt{2}(y d\varphi - x dt), \quad A^2 = 0. \quad (5.12)$$

Transforming to the $D = 11$ variables we obtain

$$g_2 = g_3 = 1, \quad \phi = \psi = \kappa = 0, \quad F_{[4]} = \sqrt{2} Vol(2) \wedge (dy \wedge d\varphi - dx \wedge dt). \quad (5.13)$$

For the different order of vector fields $\zeta_1 = \eta, \zeta_2 = \chi$ one obtains the same g_2, g_3 and zero scalars ϕ, ψ, κ but a different four-form:

$$F_{[4]} = *\sqrt{2}(dy \wedge d\varphi - dx \wedge dt), \quad (5.14)$$

where a star denotes the $D = 6$ Hodge dual. In both cases the $D = 11$ metric is a trivial smearing of the $D = 6$ metric:

$$ds_{11}^2 = ds_6^2 - dx_6^2 - \dots - dx_{10}^2. \quad (5.15)$$

6 Breaking of supersymmetry

Now let us discuss the issue of supersymmetry. As it is well-known, the Bertotti-Robinson solution preserves all the supersymmetries of $D = 4, \mathcal{N} = 2$ supergravity [29, 30]. The Bardeen-Horowitz solution (2.37) is a vacuum one, so it can be probed for $\mathcal{N} = 1$ supersymmetry. The result is negative: no geometric Killing spinors exist. Our solution (2.23) should be tested in the context of $D = 4, \mathcal{N} = 4$ supergravity, the relevant equations coming from the supersymmetric variation of the dilatino and gravitino. The variation of the dilatino leads to a purely algebraic equation, which in the case of a vanishing dilaton reads

$$(\gamma^\mu \partial_\mu \kappa + i\sqrt{2}\sigma^{\bar{\mu}\bar{\nu}} F_{\bar{\mu}\bar{\nu}}^-) \epsilon = 0, \quad (6.1)$$

where F^- is the anti-self-dual part of the Maxwell tensor.

Substituting here $\kappa = \cos \theta$ and the Maxwell tensor (2.35) one obtains the equation

$$M(\theta)\epsilon = 0, \quad M = \gamma^{\bar{\theta}} \sin \theta - (\cos \theta + i)(\sigma^{\bar{\theta}\bar{\varphi}} - i\sigma^{\bar{t}\bar{x}}). \quad (6.2)$$

The determinant $|\det M| = \sin^4 \theta$, so that there is no non-trivial solution to Eq. (6.1), i.e. the BREMDA bosonic solution breaks all the supersymmetries of $D = 4, \mathcal{N} = 4$ supergravity. Similarly, one can show that the Bertotti-Robinson endowed with NUT (4.10) is not supersymmetric either.

Now discuss the $D = 11$ embedding. Our $D = 11$ solution is related to the four-dimensional BREMDA in a non-local way, since it is obtained using dualizations in the intermediate dimensions. So a priori it is not clear whether it is non-supersymmetric in the supergravity sense.

It was shown [16] that the $D = 11$ Killing spinor equation for the 32-component Majorana spinor $\epsilon_{(11)}$ ensuring the vanishing of the supersymmetry variation of the gravitino

$$\hat{D}_M \epsilon_{(11)} + \frac{1}{288} \left(\Gamma_M^{\bar{N}\bar{P}\bar{Q}\bar{R}} - 8\delta_M^{\bar{N}} \Gamma^{\bar{P}\bar{Q}\bar{R}} \right) \hat{F}_{\bar{N}\bar{P}\bar{Q}\bar{R}} \epsilon_{(11)} = 0 \quad (6.3)$$

for the $2+3+6$ block truncation considered above corresponds to the purely geometric equation for the eight-dimensional dual:

$$(\partial_{\bar{\mu}} - \frac{1}{4} \omega_{\bar{\mu}}^{ab} \sigma_{ab}) \epsilon_8 = 0 \quad (6.4)$$

We will use the flat gamma-matrices, and $a, b, \bar{\mu}$ are the tetrad and coordinate indices respectively. Here the spin-connection $\omega_{\bar{\mu}}^{ab}$ has to be calculated for the spacetime (5.2). Therefore to explore the supersymmetry in the $D = 11$ supergravity sense we have to check whether the corresponding $D = 8$ solution admits covariantly constant spinors.

The non-zero spin-connection one-forms for the metric (5.11) read (we use tetrad indices and numbering 0, 1, 2, 3, 4 for $t, \xi, \theta, \varphi, \chi$):

$$\begin{aligned} \omega_{01} &= \cos \theta d\varphi + d\chi/\sqrt{2}, & \omega_{04} &= d\xi/\sqrt{2}, & \omega_{14} &= -\sinh \xi dt/\sqrt{2}, \\ \omega_{23} &= \cosh \xi dt - d\chi/\sqrt{2}, & \omega_{24} &= -\sin \theta d\varphi/\sqrt{2}, & \omega_{34} &= d\theta/\sqrt{2}. \end{aligned} \quad (6.5)$$

For the gamma matrices in eight dimensions one can use suitably defined tensor products of Pauli matrices. Since the spin-connection lies entirely in the $D = 6$ sector, one can suppress spinor indices relating to the transition from six to eight dimensions and use 8×8 gamma matrices and $D = 6$ spinors. A convenient choice is

$$\begin{aligned} \Gamma^0 &= i\sigma_1 \otimes 1 \otimes 1, & \Gamma^1 &= \sigma_2 \otimes 1 \otimes 1, \\ \Gamma^2 &= \sigma_3 \otimes \sigma_1 \otimes 1, & \Gamma^3 &= \sigma_3 \otimes \sigma_2 \otimes 1, \\ \Gamma^4 &= \sigma_3 \otimes \sigma_3 \otimes \sigma_1, & \Gamma^5 &= \sigma_3 \otimes \sigma_3 \otimes \sigma_2. \end{aligned} \quad (6.6)$$

The corresponding Lorentz generators are:

$$\begin{aligned} \sigma^{01} &= -\sigma_3 \otimes 1 \otimes 1, & \sigma^{04} &= \sigma_2 \otimes \sigma_3 \otimes \sigma_1, \\ \sigma^{14} &= i\sigma_1 \otimes \sigma_2 \otimes \sigma_1, & \sigma^{23} &= i1 \otimes \sigma_3 \otimes 1, \\ \sigma^{24} &= -i1 \otimes \sigma_2 \otimes \sigma_1, & \sigma^{34} &= i1 \otimes \sigma_1 \otimes \sigma_1. \end{aligned} \quad (6.7)$$

A direct substitution in the Eq.(6.4) gives a system of matrix equations which should satisfy the integrability conditions

$$R^{ab\mu\nu} \sigma_{ab} \epsilon_6 = 0 \quad (6.8)$$

where the mixed coordinate-tetrad components of the $D = 6$ Riemann tensor are introduced. Writing them as curvature two-forms $\Omega^{ab} = R_{\mu\nu}^{ab} dx^\mu \wedge dx^\nu$, one finds the following non-zero quantities

$$\begin{aligned}
\Omega^{01} &= \sin \theta d\theta \wedge d\varphi - \frac{1}{2} \sinh \xi dt \wedge d\xi, & \Omega^{02} &= \frac{1}{2} \sin \theta d\xi \wedge d\varphi, \\
\Omega^{03} &= \frac{1}{2} d\theta \wedge d\xi, & \Omega^{04} &= \frac{1}{\sqrt{2}} \cos \theta \sinh \xi dt \wedge d\varphi + \frac{1}{2} \sinh \xi dt \wedge d\chi, \\
\Omega^{12} &= \frac{1}{2} \sin \theta \sinh \xi dt \wedge d\varphi, & \Omega^{13} &= \frac{1}{2} \sinh \xi d\theta \wedge dt, \\
\Omega^{14} &= \frac{1}{\sqrt{2}} \cosh \xi dt \wedge d\xi + \frac{1}{\sqrt{2}} \cos \theta d\xi \wedge d\varphi + \frac{1}{2} d\xi \wedge d\chi, \\
\Omega^{23} &= -\sinh \xi dt \wedge d\xi + \frac{1}{2} \sin \theta d\theta \wedge d\varphi, \\
\Omega^{24} &= -\frac{1}{\sqrt{2}} \cosh \xi dt \wedge d\theta - \frac{1}{\sqrt{2}} \cos \theta d\theta \wedge d\varphi - \frac{1}{2} d\theta \wedge d\chi, \\
\Omega^{34} &= -\frac{1}{\sqrt{2}} \sin \theta \cosh \xi dt \wedge d\varphi - \frac{1}{2} \sin \theta d\varphi \wedge d\chi.
\end{aligned} \tag{6.9}$$

All ten two-forms are independent, so one obtains ten integrability conditions:

$$\sigma^{03} \epsilon_6 = \sigma^{14} \epsilon_6 = \sigma^{24} \epsilon_6 = \sigma^{34} \epsilon_6 = 0, \tag{6.10}$$

$$\begin{aligned}
(\tanh \xi \sigma^{01} + \sqrt{2} \sigma^{14}) \epsilon_6 &= 0, & (\tanh \xi \sigma^{13} + \sqrt{2} \sigma^{24}) \epsilon_6 &= 0, \\
(\cot \theta \sigma^{04} - \coth \xi \sigma^{04} + \sqrt{2} \sigma^{12}) \epsilon_6 &= 0, \\
(\sigma^{04} + 2\sigma^{23}) \epsilon_6 &= 0, & (\tan \theta \sigma^{02} - \sqrt{2} \sigma^{14}) \epsilon_6 &= 0, \\
(2\sigma^{01} - \sigma^{23} + \sqrt{2} \cot \theta \sigma^{24}) \epsilon_6 &= 0.
\end{aligned} \tag{6.11}$$

These are clearly inconsistent (inconsistent are already conditions (6.10), since sigma-matrices do not have kernels). Therefore the BREMDA geometry is not supersymmetric in the sense of $D = 11$ supergravity either.

7 Conclusion

We have presented a new solution to dilaton-axion gravity in four dimensions which is a rotating version of the Bertotti-Robinson metric. It breaks the $SO(3)$ symmetry of the latter but preserves the $SL(2, R)$ symmetry of the anti-de Sitter sector. The metric arises as the near-horizon limit of the charged rotating axion-dilaton black hole (in the theory with one vector field) and is supported by non-trivial vector and axion fields. It looks simpler than the near-horizon Kerr (or Kerr-Newman) metric due to the absence of additional angular factors, while preserving the same mixing of the azimuthal and time coordinates induced by rotation. It is important to note that the AdS sector does not factor out even asymptotically. Moreover, in contrast to the case of $AdS_2 \times S^2$, the conformal boundary is now a singular $1 + 2$ space.

The new metric was shown to be related to the usual dyonic BR solution with equal electric and magnetic charges after uplifting it to six dimensions and then coming back along a different reduction scheme. In this procedure the axion emerges via dualization of one of the Kaluza-Klein two-forms. Using a similar reasoning, we were able to find two solutions of $D = 11$ supergravity

with non-trivial four-form fields whose dimensional reduction (including dualizations at intermediate steps) gives our solution.

This solution is not supersymmetric, whether in the sense of the original $D = 4, \mathcal{N} = 4$ supergravity, or in the sense of higher dimensional embeddings. But it still looks promising from the point of view of holography. Indeed, it preserves some features of $AdS_2 \times S^2$ found also for the Kerr throat [3], and it is not plagued by superradiance as the latter. Preliminary considerations show that, in spite of the singular nature of the boundary, the asymptotic symmetry contains the Virasoro algebra [25]. Also it is likely to provide a new version of conformal mechanics of the type studied recently [31, 32]. We will discuss these issues in a separate publication.

Acknowledgements

We would like to thank K. Bronnikov, J. Fabris, L. Palacios, and S. Solodukhin for discussions. DG is grateful to LAPTH Annecy for hospitality and to CNRS for support which made possible this collaboration. His work was also supported in part by the RFBR grant 00-02-16306.

Appendix A

In this Appendix we discuss the near-horizon limit to static black hole solutions of 5-dimensional sourceless Kaluza-Klein theory, i.e. 5-dimensional vacuum Einstein gravity

$$S = - \int d^5x \sqrt{|g_5|} R_5, \quad (\text{A.1})$$

together with the assumption of a spacelike Killing vector $\partial/\partial x^5$. The 5-dimensional metric may be reduced to 4 dimensions by the Kaluza-Klein dimensional reduction

$$ds_5^2 = e^{-2\sigma/\sqrt{3}} ds_4^2 - e^{4\sigma/\sqrt{3}} (dx^5 + 2A_\mu dx^\mu)^2, \quad (\text{A.2})$$

with the reduced action

$$S = \int d^4x \sqrt{|g_4|} \left\{ -R_4 + 2\partial_\mu \sigma \partial^\mu \sigma - e^{2\sigma/\sqrt{3}} F_{\mu\nu} F^{\mu\nu} \right\}. \quad (\text{A.3})$$

The general static, NUT-less black hole solution of Kaluza-Klein theory was derived by Gibbons and Wiltshire [33], and generalized to rotating black hole solutions by Rasheed [34]. We will consider only static black holes, which depend on 3 parameters M (mass), Σ (scalar charge), Q and P (“electric” and “magnetic” charge) constrained by

$$\frac{Q^2}{\Sigma + M/\sqrt{3}} + \frac{P^2}{\Sigma - M/\sqrt{3}} = \frac{2\Sigma}{3}. \quad (\text{A.4})$$

The corresponding 5-dimensional metrics, as well as their 4-dimensional reductions, have two regular horizons provided $M^2 + \Sigma^2 - P^2 - Q^2 \geq 0$. The condition of extremality is therefore $M^2 + \Sigma^2 = P^2 + Q^2$. However for more generality we shall consider near-extremal black holes, with

$$M^2 + \Sigma^2 - P^2 - Q^2 \equiv \lambda^2 c^2 \quad (\text{A.5})$$

small.

The black hole solutions are

$$ds_5^2 = \frac{f^2}{B} dt^2 - A \left(\frac{dr^2}{f^2} + d\theta^2 + \sin^2 \theta d\varphi^2 \right) - \frac{B}{A} \left(dx^5 + \frac{2Q}{B} (r - M + M_-) dt + 2P \cos \theta d\varphi \right)^2, \quad (\text{A.6})$$

where the metric functions f , A , B are given by

$$\begin{aligned} f^2 &= (r - M)^2 - \lambda^2 c^2, \\ A &= f^2 + 2M_-(r - M) + \frac{M_-}{M} (M_+ M_- + \lambda^2 c^2), \\ B &= f^2 + 2M_+(r - M) + \frac{M_+}{M} (M_+ M_- + \lambda^2 c^2), \end{aligned} \quad (\text{A.7})$$

and

$$M_{\pm} \equiv M \pm \frac{\Sigma}{\sqrt{3}}, \quad Q^2 = \frac{M_+(M_+^2 - \lambda^2 c^2)}{2M}, \quad P^2 = \frac{M_-(M_-^2 - \lambda^2 c^2)}{2M}. \quad (\text{A.8})$$

Putting $r - M \equiv \lambda x$, we shall take the near-extremal, near-horizon limit $\lambda \rightarrow 0$ such that the two horizons $r = r_{\pm} \equiv M \pm \lambda c$ approach each other while the radial coordinate approaches the event horizon r_+ . Four-dimensional sections $\varphi = \text{const.}$ of the 5-dimensional metric (A.6) being similar in form to the rotating metric (2.11), with the electric potential A_4 playing the part of the angular velocity, to obtain a finite limit we again must first transform to a frame “nearly co-rotating” with the horizon, through a gauge transformation

$$d\bar{x}^5 = dx^5 + \frac{2QM_-}{B(0)} dt, \quad (\text{A.9})$$

with $B(0) \equiv B(r - M = 0)$, leading to the “electric” field in the new gauge

$$\bar{A}_4 = -\frac{QM_-}{B(0)B} (r - M) \left(\frac{M_+ M_-}{M} + r - M + O(\lambda^2 c^2) \right), \quad (\text{A.10})$$

and rescale both time and the fifth coordinate, through the transformations

$$r - M \equiv \lambda x, \quad \cos \theta \equiv y, \quad t \equiv \frac{\sqrt{A_0 B_0}}{\lambda} \bar{t}, \quad \bar{x}^5 \equiv \sqrt{2} P \chi, \quad (\text{A.11})$$

where $(A_0, B_0) = \lim_{(\lambda \rightarrow 0)} (A, B)$. Taking the limit $\lambda \rightarrow 0$, using the identities

$$\frac{2B_0 P^2}{A_0^2} = \frac{Q^2 A_0 M_-^2}{P^2 B_0 M_+^2} = 1, \quad (\text{A.12})$$

and relabelling the time coordinate $\bar{t} \rightarrow t$, we finally obtain the 5-dimensional near-horizon metric

$$A_0^{-2} ds_5^2 = (x^2 - c^2) dt^2 - \frac{dx^2}{x^2 - c^2} - \frac{dy^2}{1 - y^2} - (1 - y^2) d\varphi^2 - (d\chi - \sqrt{2}x dt + \sqrt{2}y d\varphi)^2. \quad (\text{A.13})$$

Again, as in the case of EMDA, all the static Kaluza-Klein black holes have the same near-horizon limit, independently of the values of the electric and magnetic charges $Q \neq 0$ and $P \neq 0$.

In (A.13) we recognize the Kaluza-Klein version of the dyonic Bertotti-Robinson solution with equal electric and magnetic charges. The vanishing of the Kaluza-Klein scalar field σ is due to the fact that the electric and magnetic fields

$$F_{14} = -F_{23} = -\frac{1}{\sqrt{2}} \quad (\text{A.14})$$

being equal in magnitude, the source term in the scalar field equation

$$\nabla^2 \sigma = -\frac{1}{2\sqrt{3}} e^{2\sqrt{3}\sigma} F^{\mu\nu} F_{\mu\nu} \quad (\text{A.15})$$

vanishes. Accordingly the isometry group is the direct product of that of the Bertotti-Robinson spacetime with the Klein circle, *i.e.* $SL(2, R) \times SO(3) \times U(1)$.

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